

QUANTIFYING CONTINGENT RELATIONS FROM DIRECT  
OBSERVATION DATA: TRANSITIONAL PROBABILITY  
COMPARISONS VERSUS YULE'S Q

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Measuring contingencies or sequential associations may be applied to a broad range of response–stimulus, stimulus–stimulus, or response–response relations. Within behavior analysis, response–stimulus contingencies have been quantified by comparing 2 transitional probabilities and plotting them in contingency space. Within and outside behavior analysis, Yule's Q has become a recommended statistic used to quantify sequential associations between 2 events. In the current paper, we identify 2 methods of transitional probability comparisons used in the behavior-analytic literature to estimate contingencies in natural settings. We compare each of these methods to the more established Yule's Q statistic and evaluate relations between each pair of indices. Advantages and disadvantages of each method are identified, with recommendations as to which approach may be most appropriate for measuring contingencies.

*Key words:* contingency, descriptive analysis, sequential analysis, transitional probability, Yule's Q

Descriptive analyses of behavior–environment interactions contribute to our understanding of human behavior as it occurs in natural contexts. Because environmental variables are not controlled or systematically manipulated, the data collected may be used only to describe, not explain, behavior patterns. However, descriptions of behavior–environment interactions may guide subsequent experimental analyses that do allow the identification of functional relations. For example, higher likelihoods of preschool student compliance following certain types of teacher instructions (e.g., Ndoro, Hanley, Tiger, & Heal, 2006) may lead to evaluating the effects of instructions on student compliance. Associations between peer-

directed interactions by young children with autism and immediate social consequences (e.g., Boyd, Conroy, Asmus, & McKenney, 2011) may lead to experimental evaluations of interventions designed to increase social interactions by manipulation of social consequences. Alternatively, descriptions of behavior–environment interactions may be useful following experimental analyses. After social reinforcers for problem behaviors are identified via functional analyses, descriptive data may be used to estimate contingencies between problem behaviors and corresponding reinforcers in naturalistic settings (e.g., Borrero & Vollmer, 2002).

Descriptive analyses of response–stimulus relations (e.g., McKerchar & Thompson, 2004; Symons, Hoch, Dahl, & McComas, 2003; R. H. Thompson & Iwata, 2001; Vollmer, Borrero, Wright, Van Camp, & Lalli, 2001), response–response relations (e.g., Marion, Touchette, & Sandman, 2003; Matthews, Shimoff, & Catania, 1987), and stimulus–stimulus relations (e.g., Gibbon, Berryman, & Thompson, 1974) are abundant in the literature yet vary in the methods used to quantify these relations. The variety of methods used to describe relations

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between responses, environmental stimuli, or both highlight a lack of consensus as to which methods may be most appropriate. For the remainder of this paper, we use the term *response–stimulus relations* to discuss contingencies or associations between two events. The relations described, however, may be applied to any number of response–response, stimulus–stimulus, or stimulus–response sequences.

Evaluation of differences between transitional probabilities to identify contingencies between responses and environmental stimuli is one type of approach commonly used in the behavior-analytic literature (e.g., Anderson & Long, 2002; Luczynski & Hanley, 2009; Martens, DiGenaro, Reed, Szczech, & Rosenthal, 2008; Watson, 1997). A transitional probability is the proportion of instances of one event that are followed by another event in sequence (Yoder & Symons, 2010). Transitional probability comparisons evolved from behavior-analytic conceptualizations of operant contingencies as differences between two transitional probabilities. One way operant contingencies have been defined is the difference between the transitional probabilities of a stimulus (a) following and (b) not following a target response (Catania, 2006, 2007; Gibbon *et al.*, 1974; Hammond, 1980). That is, the transitional probability of a stimulus following a target response is compared to the transitional probability of a stimulus in the absence of a target response. Applied to a descriptive analysis of teacher instructions and student compliance, for example, the probability of compliance following embedded directives is compared to the probability of compliance in the absence of embedded directives. A positive contingency is identified if compliance were more likely to occur in the presence than in the absence of embedded directives. A negative contingency is identified if compliance were less likely to occur in the presence than in the absence of embedded directives.

Whereas transitional probability comparisons are grounded in operant theory, a second method

of quantifying response–stimulus relations has been established both within and outside the field of behavior analysis. *Yule's Q* (Yule & Kendall, 1957) is a statistic designed to quantify categorical data associations. Categorical data analysis methods are used to describe associations between nominally scaled (i.e., categorical) variables, including those with binary outcomes such as present and absent. Yule's *Q* is the recommended method for statistically quantifying sequential associations between two events (Yoder & Feurer, 2000; Yoder, Short-Meyerson, & Tapp, 2004) and has been suggested as a method of quantifying response–stimulus contingencies (McComas *et al.*, 2009). Yule's *Q* ranges from  $-1$  to  $1$ , with positive values indicating observed sequential frequencies greater than expected by chance, and negative values indicating observed sequential frequencies less than expected by chance (Yoder & Symons, 2010). Applied to the example descriptive analysis of teacher instruction type and student compliance, a positive Yule's *Q* index indicates that compliance followed embedded directives more than would be expected by chance alone. A negative Yule's *Q* index indicates that compliance followed embedded directives less often than would be expected by chance alone.

Although each method appears to be promising, the literature contains no comparison of Yule's *Q* and transitional probability differences as they are used to calculate response–stimulus relations from direct observation data. In addition, Yule's *Q* and various transitional probabilities have been defined in different terms, making it difficult to understand the relations between each measure. The goals of the current paper are (a) to describe two potentially valuable transitional probability approaches to quantify operant contingencies, (b) to compare each of these approaches to a well-established statistic of sequential association (i.e., Yule's *Q*), (c) to identify potential advantages and disadvantages of each method, (d) to evaluate the relations between these

indices using simulated data, and (e) to provide recommendations as to which method may be most useful.

### TRANSITIONAL PROBABILITY COMPARISONS AND CONTINGENCY SPACE

Operant contingencies have been defined as the difference between the transitional probability of a stimulus following a target response and the transitional probability of a stimulus not following a target response (Catania, 2007). These transitional probabilities may be compared by plotting each probability in contingency space. Contingency space is a coordinate system in which probabilities of one event in the presence and the absence of another event are plotted (Matthews et al., 1987). Contingency space analysis (CSA) has been used to identify contingent relations between conditioned and unconditioned stimuli in a classical conditioning framework (Gibbon et al., 1974), between verbal responding and nonverbal responding (i.e., correspondence between saying and doing; Matthews et al., 1987), and, more recently, between problem behaviors and environmental stimuli (e.g., Eckert, Martens, & DiGennaro, 2005; Martens et al., 2008; Martens, Gertz, de Lacy Werder, & Rymanowski, 2010; Reed, Luiselli, Morizio, & Child, 2010).

CSA involves plotting the transitional probability of a stimulus given a target response along the  $y$  axis and plotting the transitional probability of a stimulus given the absence of a target response along the  $x$  axis. The following types of relations between target responses and environmental stimuli may be described via CSA and are shown in Figure 1. In contingency space, points located above the graphic diagonal represent positive contingencies (i.e., the probability of the stimulus is higher in the presence of the target response than in its absence; Hammond, 1980). Points located below the graphic diagonal represent negative contingencies (i.e., the proba-

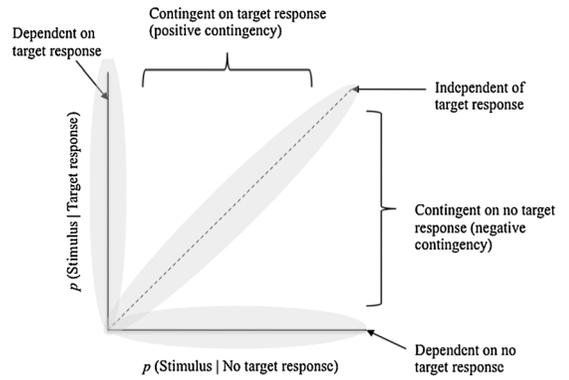


Figure 1. Visual representation of relations between target responses and stimuli in contingency space.

bility of the stimulus is lower in the presence of the target response than in its absence; Hammond, 1980). Points on the  $y$  axis and  $x$  axis represent the degree of dependent relations. When plotted on the  $y$  axis, the stimulus occurs exclusively in the presence of the target response, and when plotted on the  $x$  axis, the stimulus occurs exclusively in the absence of the target response, yet each probability may vary from 0 to 1. Points located on, or near, the diagonal line between the  $x$  and  $y$  axes indicate that the probability of the stimulus is the same or similar given the presence or absence of the target response (Catania, 2007).

The location of points in contingency space allows a visualization of contingent, dependent, and independent relations between responses and environmental stimuli. In addition to a visual comparison of transitional probabilities in contingency space, the difference between the two transitional probabilities may be used as an index of contingency strength (e.g., Hammond, 1980). This difference score represents the distance from the diagonal of independence in contingency space; thus, presumably, the farther the point lies from the diagonal, the stronger the contingency. To plot transitional probabilities in contingency space, these probabilities must first be calculated using values from a  $2 \times 2$  contingency table.

### The $2 \times 2$ Contingency Table

Indices of contingency strength and sequential association can be calculated from the four cell values of  $2 \times 2$  contingency tables used to organize raw sequential data. To allow such organization, the data must meet several requirements. First, data must be collected using timed event or interval methods that preserve the temporal integrity of events. Second, each event of interest (e.g., response and stimulus) must consist of mutually exclusive categories (e.g., teacher instruction is either present or absent; student compliance is either present or absent). Third, each event must be coded independent of the other event (i.e., the response may be coded in the absence of the stimulus, and the stimulus may be coded in the absence of the response). For example, student compliance must be able to occur in the absence of a particular type of teacher instruction, and this type of teacher instruction must be able to occur in the absence of student compliance. A sequential analysis of general teacher instructions and student compliance would fail to meet this criterion if the occurrence of compliance could be coded only following the occurrence of an instruction. Fourth, all relevant coded units (e.g., all intervals for interval methods of data collection) must be included to create an exhaustive coding space. This last requirement of  $2 \times 2$  tables is relevant to several points to which we will return.

Coded units that represent the presence or absence of target responses and stimuli are tallied into the cells of the  $2 \times 2$  contingency table. Coded units may be events, time units (e.g., seconds), or intervals. Each coded unit from an observation is tallied in one of four cells. As depicted in Table 1, Cell A includes all coded

units in which both the target response and stimulus are present. Cell B includes all coded units in which the target response is present but the stimulus is absent. Cell C represents coded units in which the target response is absent but the stimulus is present, and Cell D represents coded units in which both the target response and stimulus are absent.

Construction of the  $2 \times 2$  contingency table requires defining the absence of a response or stimulus to determine whether one event follows another in sequence. When coded units consist of intervals or time windows, a particular interval duration or time window must be selected as a boundary for whether a response or stimulus will be coded as occurring. For example, 15-s intervals may be selected such that if a stimulus does not occur within a 15-s interval, the interval is defined as *stimulus absent*. Or, a 10-s window may be selected such that if a stimulus occurs 9 s after a target response, the stimulus is coded as following the response in sequence and the window will be tallied in Cell A. If the stimulus occurs 11 s after a target response, however, the stimulus is coded as absent and the window will be coded in Cell B. Clearly, defining how the  $2 \times 2$  contingency tables are constructed has an arbitrary component. To the extent feasible, these definitions should be based on the type of response–stimulus, stimulus–stimulus, or response–response relations under investigation. For example, 1-s intervals may be appropriate when describing associations between unconditioned and conditioned stimuli in a classical conditioning framework, whereas 15-s intervals may be appropriate when describing associations between teacher instructions and student compliance.

The computational formula for each transitional probability may be stated in terms of the four cell values of the contingency table. The transitional probability of a stimulus given a target response is calculated as follows and is shown in Table 2. The number of coded units containing the target response followed by the

Table 1  
Construction of the  $2 \times 2$  Contingency Table

	Stimulus present	Stimulus absent
Target response present	A	B
Target response absent	C	D

Table 2  
Formulae for Transitional Probability Comparisons and Yule's Q

Measure	Formula
Exhaustive CSA	$\frac{A}{A+B}, \frac{C}{C+D}$
Nonexhaustive CSA	$\frac{A}{A+B}, \frac{C}{A+C}$
Yule's Q	$\frac{AD-BC}{AD+BC}$

stimulus (A) is divided by the total number of coded units containing the target response (A + B). The second transitional probability, the probability of a stimulus given the absence of a target response, may be calculated in one of two ways. We will describe one method of calculation here, then return to a second method below. In previous studies with CSA, the probability of a stimulus given the absence of a target response was calculated as the number of coded units containing the stimulus without a preceding target response (C) divided by the total number of coded units in which the target response was absent (C + D). This method includes all four cells of the contingency table, and thus preserves an exhaustive coding space.

The first transitional probability controls for the total occurrence of the target response by dividing the A cell value by the total number of coded units including the target response. By *controls for*, we mean the effect of a confounding variable on the outcome of interest is minimized. In this case, the confounding effect of the total occurrence of the response on the index of contingency or sequential association is minimized. If a confounding variable is adequately controlled, a zero correlation should be identified between this variable and the outcome measure. The second transitional probability controls for the nonoccurrence of the target response in a similar fashion. Neither of the transitional probabilities is generated by dividing the total number of coded units including the stimulus. Thus, it is reasonable to question whether the occurrence of the stimulus is controlled. In addition, the extent to which these transitional

probabilities are successful in controlling for base rates of the target response and stimulus has not yet been examined in the literature. *Base rates* refer to event counts that are divided by the total number of coded units. That is, base rates reflect counts conditional on total observation time, total coded intervals, or the total number of coded events.

Recent studies by Luczynski and Hanley (2009, 2010) have included a different comparison transitional probability from that used in CSA research. Instead of dividing the number of coded units that contain the stimulus without a preceding target response (C) by the total number of coded units in which the target response was absent (C + D), this number (C) was divided by the total number of coded units in which the stimulus was present (A + C). To our knowledge, this contrast probability has not been included in previous CSA research, but it appears to be a viable alternative, because it preserves the contingent, dependent, and independent relations that may be described in contingency space and appears to control for the total occurrence of both the target response and stimulus by dividing key cell values by the total coded units that include each event. However, the extent to which this method successfully controls for base rates of each event is also unclear and has not been evaluated in the literature. In addition, because this method includes only three of the four cells of the contingency table, the coding space is no longer exhaustive and controlling for base rates may not be possible.

*The D Cell Problem*

When comparing each of the above transitional probability strategies, the question becomes whether it is important to include the D cell (Table 2) in calculating contingencies. The traditional CSA approach, from here on referred to as the *exhaustive* CSA, does include the D cell, whereas the latter approach, from here on referred to as the *nonexhaustive* CSA, does not include the D cell. The D cell represents all coded units in

which neither the target response nor the stimulus occurs. As commonly practiced in descriptive analyses of behavior–environment interactions in natural settings, intervals or time windows are the coded units. When intervals or time windows are the coded units, the D cell represents the passage of time without the occurrence of either event of interest. Conceptually, one might describe the D cell as providing a temporal context for the remaining cell values.

A potential problem of including the D cell, however, is that the mere passage of time alters the strength of the contingency index. For example, consider two classroom observations of teacher-embedded directives and student compliance. During both observations, 10 intervals are coded as including both an embedded directive and compliance ( $A = 10$ ), 5 intervals are coded as including embedded directives with no compliance ( $B = 5$ ), and 5 intervals are coded as including compliance without embedded directives ( $C = 5$ ). However, the first observation was shorter, and 20 intervals remained without the occurrence of either event ( $D = 20$ ). The second observation was longer, and 50 intervals remained without the occurrence of either event ( $D = 50$ ). Under these conditions, the longer observation yields a greater difference in transitional probabilities ( $.67-.09 = .58$ ) than the shorter observation ( $.67-.20 = .47$ ) when the exhaustive CSA is used. When the nonexhaustive CSA is used, each observation yields the same difference ( $.67-.33 = .34$ ). That is, all other cell values being equal, longer sessions produce greater transitional probability differences when the D cell is included. Currently, the field lacks consensus as to whether this consequence of including the D cell is consistent with the concept of contingency.

### YULE'S Q

Although Yule's Q has been applied in the context of measuring response–stimulus contingencies (e.g., McComas *et al.*, 2009; Symons

*et al.*, 2003; Taylor & Oliver, 2008), this statistic is used more broadly to determine whether the occurrence of one observed event is associated with the subsequent occurrence of another observed event within one or more observations (Yoder & Symons, 2010). Yule's Q is based on the odds ratio, which is a statistic used to describe the association between two categorical variables. The Yule's Q statistic is a linear transformation of the odds ratio (Reynolds, 1984), meaning that the two descriptive statistics represent the same variable but have different units of measurement. In comparison to the odds ratio's range of zero to infinity, the transformed Yule's Q statistic ranges from  $-1$  to  $1$  and is therefore easier to interpret.

Yule's Q values  $<0$  to a nontrivial extent indicate negative sequential associations, which are interpreted to mean the stimulus (second event) followed the response (first event) less often than would be expected by chance. Values  $>0$  to a nontrivial extent indicate positive sequential associations, which are interpreted to mean the stimulus followed the response more often than would be expected by chance. (The distinction between *trivial* and *nontrivial* sequential associations may be operationalized according to the benchmark for a "small" effect size described below.) Values at or near 0 indicate no sequential association. Near-0 sequential associations are interpreted to mean the stimulus followed the response as often as would be expected by chance alone.

Yule's Q is also an effect-size measure, meaning that this statistic not only indicates the direction of association (i.e., positive, negative, zero) but also estimates the strength or magnitude of association between two variables (Reynolds, 1984). Established benchmarks for small, moderate, and large effects (.20, .43, and .60, respectively) are used to interpret Yule's Q values and are based on those provided for the odds ratio (Rosenthal, 1996). In relation to identifying behavior–environment associations, a Yule's Q value of .65 indicates a large positive sequential association, and thus a stronger association than a

Yule's Q value of .23. Under these conditions, the stimulus that yields the "large" effect-size estimate may be described as reflecting a stronger response-stimulus contingency compared to the stimulus that yields the "small" effect-size measure. Positive response-stimulus associations or contingencies, however, would be identified for both stimuli. That is, Yule's Q values of .23 and .65 each suggest a sequential frequency greater than expected by chance.

#### *The 2 × 2 Contingency Table*

Yule's Q is calculated from the same 2 × 2 contingency table values and shares the same data requirements identified for transitional probability comparisons. The formula for Yule's Q is depicted in Table 2. Because Yule's Q incorporates the multiplication of cell diagonals rather than the sums of rows or columns (i.e., marginals), this metric controls not only for the base rates of target responses and stimuli but also for the chance association of the response-stimulus sequence (Reynolds, 1984). However, the multiplication of diagonals also results in uninterpretable Yule's Q values when both cells in either row or column have values of 0. In such cases, Yule's Q is undefined due to a value of 0 in the denominator. In addition, if any of the four cells has a value of 0, Yule's Q automatically reaches its maximum or minimum value. A strategy used to address this issue, known as Yate's correction, is to add a value of .5 to all cells when any single cell contains a value of 0 (Reynolds, 1984). This adjustment, however, still produces extreme Yule's Q values.

Additional recommendations regarding the interpretability of Yule's Q relate to the question of how much data are sufficient to yield interpretable Yule's Q indices. Specifically, contingency tables that produce expected frequencies of >5 have been identified as sufficient for interpretation (Wickens, 1993; Yoder & Symons, 2010). Expected frequencies for each cell are calculated as the sum of the cell's row multiplied by the sum of the cell's column and

dividing this quotient by the sum of all four cell values. Tables with expected frequencies of <5 are referred to as sparse tables and are considered insufficient to produce interpretable Yule's Q values. Depending on the base rates of the target responses and environmental stimuli being measured, the likelihood of obtaining sparse tables, and thus uninterpretable Yule's Q indices, varies.

#### COMPARISON OF CSA AND YULE'S Q

Comparison of CSA transitional probabilities and Yule's Q presents an interesting challenge, because each method reflects a distinct construct with potentially overlapping meanings (i.e., contingency vs. sequential association) and each originates from distinct, although not incompatible, schools of thought. Comparison of transitional probabilities via CSA is a method of identifying and quantifying contingent relations as defined by principles of operant conditioning. In contrast, Yule's Q is a statistic designed to quantify the magnitude of categorical data associations that are not influenced by the base rates of the component events. Each approach also differs in terms of how the data are interpreted. Transitional probability comparisons are interpreted through visual analysis of the departure of points from the diagonal of independence and represent the difference between two transitional probabilities. Yule's Q is interpreted by its algebraic sign and the degree to which it departs from 0.

#### *Potential Advantages of Yule's Q*

As an index of sequential association, a primary advantage of Yule's Q is that it controls not only for base rates of each event but also for the chance association between the two-event sequence. For example, if both the target response and stimulus occur very frequently throughout an observation, the likelihood of the stimulus following a target response by chance increases. The formula for Yule's Q takes into account this higher likelihood

of a chance association, and the resulting metric reflects only the sequential association beyond that expected by chance.

A second advantage of Yule's *Q* is that it represents an effect size. That is, Yule's *Q* is an estimate of the magnitude of association between two variables. This means the value of Yule's *Q* not only indicates the direction of association between a response and a stimulus but also the strength of association. The strength of response–stimulus relations is relevant to the quantification of contingency because in many cases, contingencies between multiple responses, stimuli, or both may be evaluated and compared. Returning to the example descriptive analysis of teacher directives and student compliance, associations between multiple types of teacher directives and student compliance may be compared to determine whether student compliance may be more likely to follow one type of teacher directive than another. For example, in addition to identifying positive sequential associations between embedded directives and compliance and between “don't” commands and compliance, Yule's *Q* values also may reveal a stronger positive association between embedded directives and compliance than between “don't” commands and compliance. Estimating the strength of these associations allows increased precision in describing contingencies in naturalistic settings. It is important to note, however, that a strong positive sequential association between embedded directives and compliance may not be used to infer a causal or functional relation between the two events.

Finally, because Yule's *Q* is based on a well-established index of categorical association, the same guidelines apply to its computation as apply to other indices of association. Guidelines have been offered, for example, to determine whether contingency tables have sufficient or insufficient data to produce interpretable Yule's *Q* values. Although it is accepted that to produce reliable indices of contingency, transitional probability comparisons must also be calculated from a

sufficient sample of descriptive data, but no guidelines exist as to what is considered sufficient.

#### *Potential Advantages of Transitional Probability Comparisons*

Aside from being consistent with a behavior-analytic concept of contingency and method of data interpretation (i.e., visual analysis), transitional probability comparisons in contingency space have other potential advantages over Yule's *Q*. Although single effect-size measures efficiently convey the magnitude of an association, transitional probability comparisons may provide more descriptive information than a single index of association. For example, consider the following  $2 \times 2$  contingency table values:  $A = 20$ ,  $B = 80$ ,  $C = 0$ , and  $D = 100$ . These data represent a dependent relation between a target response and stimulus (i.e., the stimulus occurred only in the presence of the target response and never in its absence). The transitional probability of the stimulus given the target response is .20. The transitional probability of the stimulus in the absence of the target response is 0, regardless of which contrast probability is used. Using the Yate's correction, the Yule's *Q* value would be .96, indicating a strong positive sequential association.

Although both methods may lead to the identification of a positive association or contingency between the two events, the transitional probability approach also indicates that rather than a 1:1 correspondence, every fifth instance of the response, on average, was followed by the stimulus. This information is relevant to defining response–stimulus contingencies. Potential schedules of stimulus delivery in natural environments may be approximated and in some cases incorporated into subsequent experimental analyses. These approximated schedules of stimulus delivery, however, are apparent only by identifying each transitional probability separately and not by calculating the difference between the two transitional probabilities. That is, the truncation of two transitional probabilities into one difference score does result in a loss of information. An

advantage of transitional probability comparisons via CSA is that each transitional probability remains transparent via visual analysis of contingency space.

The CSA shown in Figure 2, for example, not only indicates a positive contingency but also allows the identification of each individual transitional probability. Based on transitional probabilities of .50 and .25, we estimate that the stimulus followed every other response on average (.50) but also occurred in the absence of responding (.25). We might also describe the stimulus as having been twice as likely to occur after the response than in the absence of the response. This information would not be available from a single estimate of sequential association or contingency strength. The difference between these transitional probabilities (i.e.,  $.50 - .25 = .25$ ), however, may still be used to estimate the strength of the overall contingency.

Another potential advantage is that a value of 0 in a cell of the contingency table does not preclude the interpretation of transitional probability comparisons. In the example table described above, the 0 in Cell C would lead to the identification of a dependent relation between the target response and stimulus. Of course, any

descriptive analysis will be useful only to the extent that it samples sufficient data. When response–stimulus contingencies are measured, the sufficiency of data will depend not only on total observation time but on how often each coded event occurs (Martens et al., 2008). However, if data were considered sufficient (e.g., a 2-hr observation in which many instances of both the target behavior and the stimulus were coded), a value of 0 in a cell would be informative. For example, a 0 in Cell A (i.e., number of intervals during which the target response was followed by the stimulus) may indicate that the stimulus is dependent on the absence of the target response, in which case a negative contingency would be identified.

*Potential Advantages and Disadvantages of Including the D Cell in CSA*

As noted previously, two different transitional probability comparisons may be made within CSA. The exhaustive CSA method includes the D cell; the nonexhaustive CSA method does not. By including the D cell, we preserve an exhaustive coding space by including all coded units in the analysis. For example, for a 60-min observation session in which a continuous 15-s partial-interval data-collection method is used, all 240 intervals would be included in calculations of contingency. Without the D cell, all coded intervals in which neither response nor stimulus occurred would be excluded from the analysis. An exhaustive coding space is a requirement for the calculation of sequential associations (Yoder & Symons, 2010). However, it is less clear whether an exhaustive coding space is a necessary condition for the identification of contingencies via transitional probability comparisons.

The D cell provides a context in which the number of coded units with a response or stimulus may be interpreted. That is, a count of 10 target responses in 2 min may mean something quite different than a count of 10 target responses in 2 hr. When coded units are time windows or intervals, inclusion of the D cell

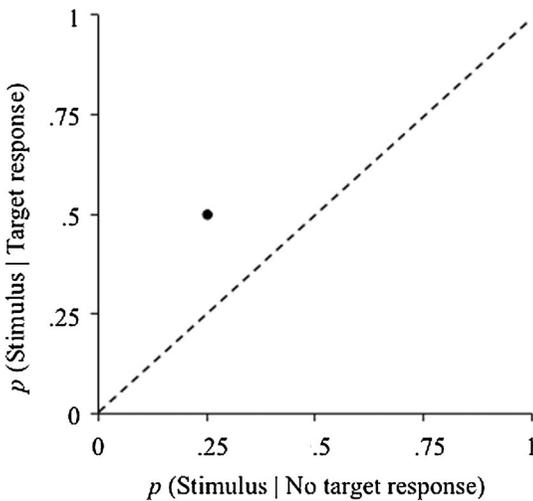


Figure 2. Example transitional probability comparison plotted in contingency space.

provides an accurate record of the observation time. If one is making comparisons between observations of varying durations, for example, accounting for time in which neither response nor stimulus occurred may make these observations more comparable. On the other hand, it is questionable whether the passage of time alone should increase the magnitude of the contingency index. Furthermore, the need for an exhaustive coding space is less clear for cases in which response–stimulus relations from the same observation session are evaluated (as opposed to response–stimulus relations across sessions of varying duration).

The primary advantage of the nonexhaustive CSA is that the total occurrence of the stimulus (i.e., second event), in addition to that of the response (i.e., first event), is controlled in an attempt to control for base rates of each event. However, the total occurrence of a response or stimulus is not identical to the base rates of each event. Base rates are calculated by dividing a total occurrence by the total number of coded units (e.g., duration of observation). The potential advantage of controlling for the base rates of both responses and stimuli seems to be of particular relevance when a variety of event types, each with varying base rates, are evaluated in a single descriptive analysis. For example, a classroom descriptive analysis may include data collection on three types of teacher instructions and two topographies of student behavior (e.g., disruptions and compliance). As is commonly the case, each type of teacher instruction and student behavior may be expected to occur with varying frequencies. Under these conditions, an index of contingency that directly controls for base rates of each event seems to be critical. When the base rate of behaviors greatly influences an index that is thought to convey contingency strength, investigators may mistakenly interpret high scores as an indication of strong contingencies when the score primarily reflects a high rate of occurrence of one or both events. For example, in an observational session in which student disruptions occur

frequently, disruptions will likely occur following embedded directives by chance.

#### EVALUATION OF RELATIONS BETWEEN TRANSITIONAL PROBABILITY DIFFERENCES AND YULE'S Q

To better understand the relations between each transitional probability approach and Yule's Q, we assessed the extent to which each pair of indices corresponded in terms of the direction and strength of the sequential association or contingency. To assess these relations, we created 200 contingency tables, the contents of which were randomly generated numbers between 0 and 100 and sampled from a uniform distribution. A uniform distribution (i.e., a type of probability distribution in which all values are equally likely), as opposed to a normal distribution (i.e., a bell-shaped probability distribution in which values near the mean are more likely to be sampled than those approaching the extremes), was selected to increase the likelihood of sampling values that produce full ranges of each index. For all contingency tables, we calculated transitional probability differences from the exhaustive CSA approach, transitional probability differences from the nonexhaustive CSA approach, and Yule's Q statistics. Three graphs were created by plotting each index against another. For each graphic comparison, Pearson product-moment correlation coefficients ( $r$ ) were calculated to estimate the linear relation between each pair of indices (B. Thompson, 2006). The linear relation between two indices may be interpreted as the extent to which the indices correspond. A perfect positive linear relation ( $r = 1.0$ ) would indicate 1:1 correspondence, such that as one index increases in value, the other index increases in value in a linear fashion.

#### *Yule's Q and the Exhaustive CSA*

Figure 3 (top left) depicts the relation between Yule's Q and the exhaustive CSA ( $A/A + B - C/C + D$ ). Perfect correspondence between any two indices is identified if the data points formed

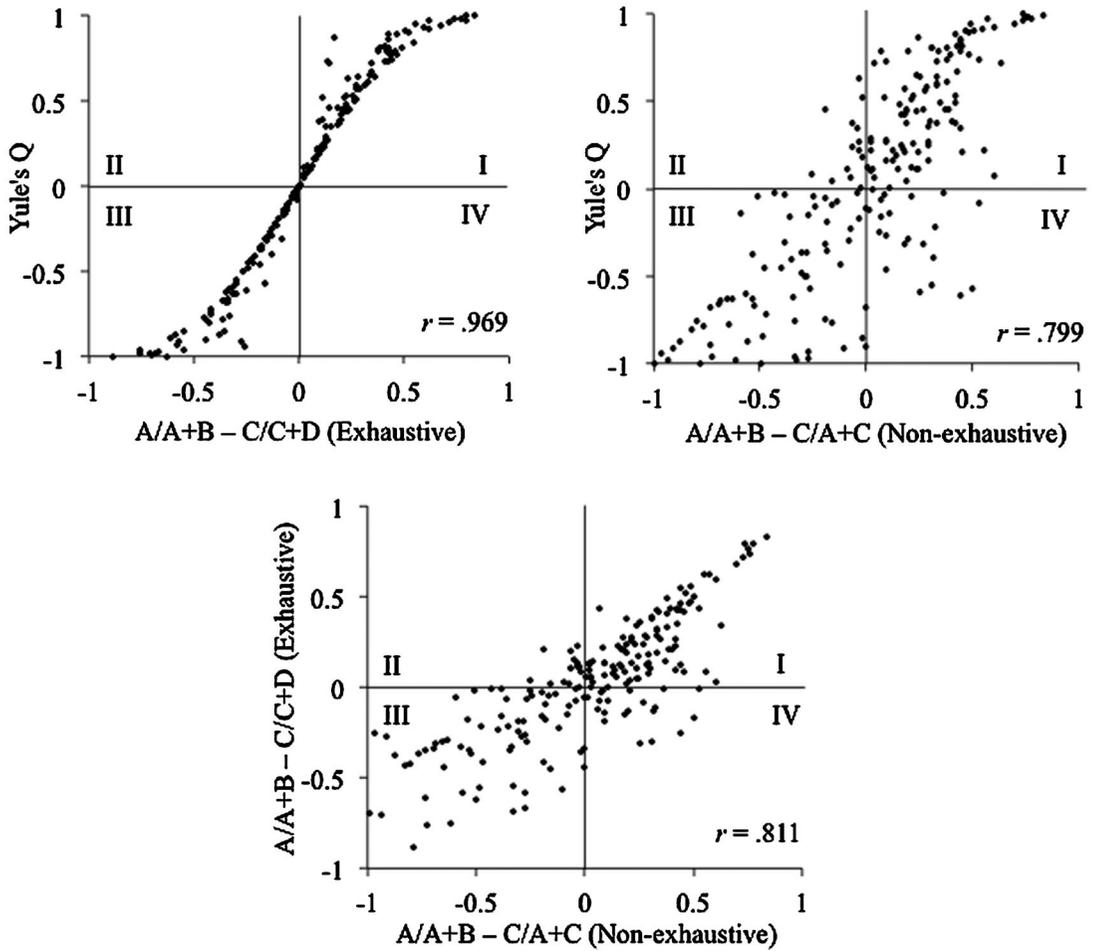


Figure 3. Relations between Yule's Q and exhaustive CSA index (top left), Yule's Q and nonexhaustive CSA index (top right), and exhaustive and nonexhaustive CSA indices (bottom).

a straight diagonal line from the origin (-1.0, -1.0) to the maximum values (1.0, 1.0) of the graph. The extent to which the data points approximate this line allows a visual interpretation of correspondence between indices. When all data points are confined to the first and third quadrants, we interpret this pattern to mean that the two indices always correspond in terms of the direction of association. Data points located in the second or fourth quadrants indicate instances in which one index produces a positive association, whereas the other index produces a negative association. As is evident in Figure 3 and by the

nature of each formula, Yule's Q and the exhaustive CSA always correspond in terms of the direction of association. A positive Yule's Q value will always correspond with a positive contingency index; a negative Yule's Q value will always correspond with a negative contingency index. This graph also shows variation in transitional probability differences at the extreme values of Yule's Q, as seen from the curvature of the data points in the first and third quadrants. This variation is likely due to the effects of sparse tables driving Yule's Q to extreme values. For example, the data points that approach each

extreme Yule's Q value (-1 and 1) but approach the midpoint of the exhaustive CSA index (0) depict contingency tables with a near-0 value in one of the cells. Although these two indices show strong linear dependence ( $r = .97$ ), the relation suggests that in a subset of cases, information from transitional probability differences may be lost with Yule's Q.

#### *Yule's Q and the Nonexhaustive CSA*

Figure 3 (top right) depicts the relation between Yule's Q and the nonexhaustive CSA ( $A/A + B - C/A + C$ ). Although there is a positive linear association between these two indices, it is clear that without the D cell, correspondence between transitional probability differences and Yule's Q decreases. In fact, the graph depicts several data points that indicate a lack of correspondence in the direction of association. For example, all data points located in the second (top left) and fourth (bottom right) quadrants of contingency space reflect cases of noncorrespondence. Data points in the second quadrant represent contingency tables in which the D cell was high relative to other cell values, producing a positive Yule's Q but a negative nonexhaustive CSA index. Data points in the fourth quadrant represent contingency tables in which the D cell was low relative to other cell values, producing a negative Yule's Q but a positive nonexhaustive CSA index. Overall, it appears that Yule's Q and the nonexhaustive CSA index are less likely to correspond for cases in which the D cell is high or low relative to the other cells in the contingency table.

#### *Exhaustive and Nonexhaustive CSA Indices*

Based on the formulae used to calculate each transitional probability difference, we know the distinction between the exhaustive and nonexhaustive CSA indices is the denominator of the second or contrast probability. Specifically, the denominators differ in terms of whether the value of A or D is added to C. This means, for cases in which D is greater than A, the exhaustive CSA

index will be larger than the nonexhaustive CSA index. This is because the C + D denominator will be larger than the A + C denominator, producing a smaller contrast probability and therefore a larger difference score. In relation to the example descriptive analysis of embedded directives and compliance, if the number of coded units with neither embedded directives nor compliance (D) is greater than the number of coded units with both embedded directives and compliance (A), the exhaustive CSA method will produce a stronger index of contingency than the nonexhaustive CSA method. Similarly, for cases in which A is greater than D, the exhaustive CSA index will be smaller than the nonexhaustive CSA index. That is, if the number of coded units with both embedded directives and compliance (A) is greater than the number of coded units with neither embedded directives nor compliance (D), the exhaustive CSA method will produce a weaker index of contingency than the nonexhaustive CSA method. The single condition in which the exhaustive and nonexhaustive CSA indices would be equal is when the A cell value (e.g., number of coded units with both events) is equal to the D cell value (e.g., number of coded units with neither event).

Figure 3 (bottom) depicts the relation between the two transitional probability differences. Again, although there is a positive linear association between these indices, there is variation in terms of the extent to which the two indices of contingency correspond. In addition, points located in the second and fourth quadrants of contingency space indicate a lack of correspondence in the direction of the contingency. The data points located in the second quadrant represent contingency tables in which the D cell was high relative to the A cell, producing a positive exhaustive CSA index and a negative nonexhaustive CSA index. The data points located in the fourth quadrant represent contingency tables in which the D cell value was low relative to the A cell, producing a negative exhaustive CSA index and a positive

nonexhaustive CSA index. This pattern suggests that a lack of correspondence is more likely when D cell values are high or low relative to A cell values.

*Changes in Relations Depending on Base Rates of the Target Response (First Event)*

We constructed additional sets of contingency tables to assess whether relations between indices change when base rates of the target response (first event) are high or low. To approximate conditions in which base rates of the target response are high, Cells A and B were generated as

random numbers between 75 and 100, and Cells C and D were generated as random numbers between 0 and 25. To approximate conditions in which base rates of the target response are low, the same set of contingency table values were rearranged such that Cells A and B were those generated as random numbers between 0 and 25 and Cells C and D were those generated as random numbers between 75 and 100.

Figure 4 (left) shows the same degree of a near-perfect linear relation between Yule's Q and the exhaustive CSA index when base rates of the target response are high or low. Figure 4 (right),

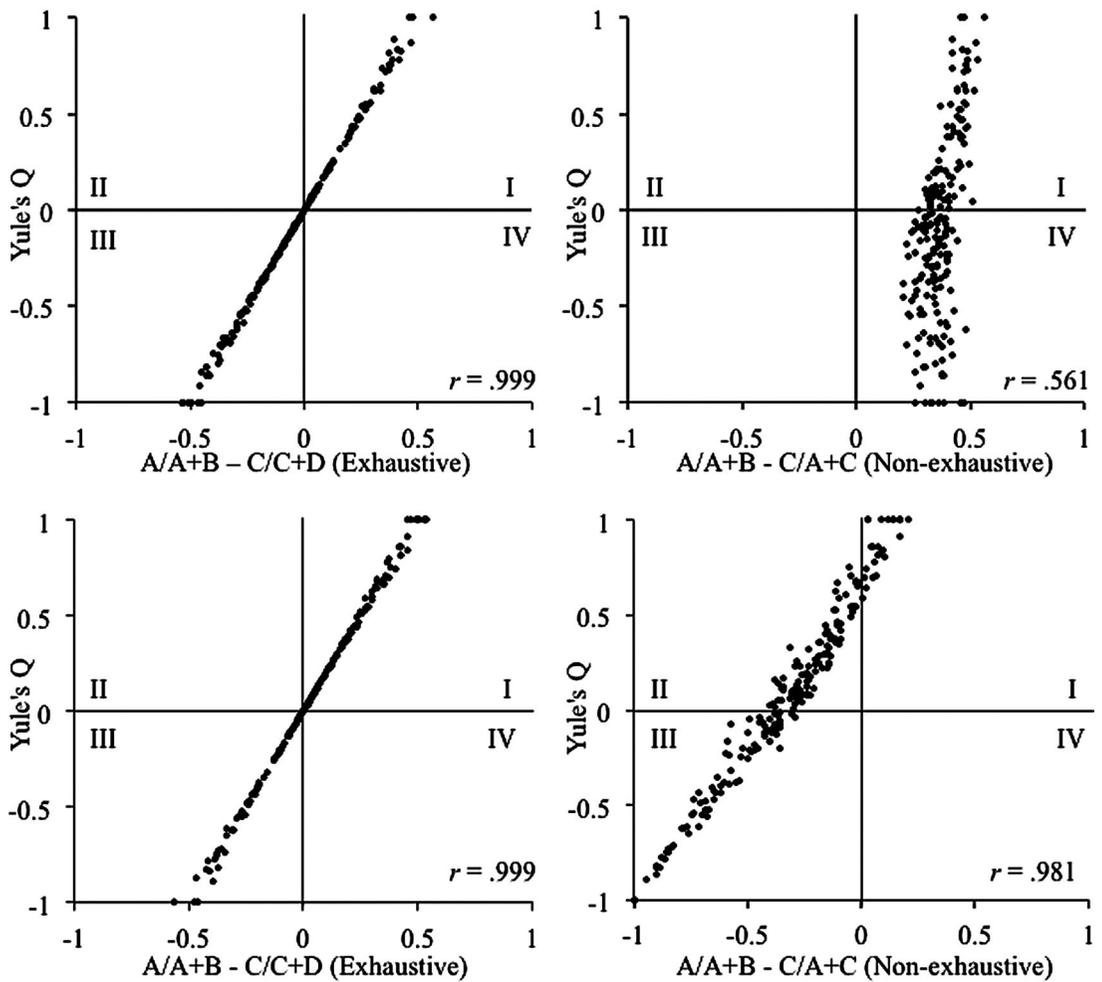


Figure 4. Relations between Yule's Q and exhaustive CSA index (left) and between Yule's Q and nonexhaustive CSA index (right) for high base rates of response (top) and low base rates of response (bottom).

however, shows a change in the relation between Yule's Q and the nonexhaustive CSA index, the nature of which depends on base rates of the target response. When base rates of the target response are high, the nonexhaustive CSA index shows a positive bias (i.e., a central tendency above zero). We refer to a nonzero central tendency as bias because the cell values are generated randomly, with the only constraint being the overall occurrence of the target response. Thus, the central tendency of the resulting contingency and sequential association indices should be zero. When the base rates of the target response are low, however, the nonexhaustive CSA index shows a negative bias (i.e., a central tendency below zero). Similar patterns also were identified between each transitional probability approach. Compared to the exhaustive CSA index, the nonexhaustive CSA index showed a positive bias when base rates of the response were high and showed a negative bias when base rates of the response were low (data are available from the first author). These biases are due to the relation between the A and D cell values. The data generated to approximate high base rates of the target response included low D cell values and high A cell values, which decreases Yule's Q or the exhaustive CSA index compared to the nonexhaustive CSA index. Similarly, the data generated to approximate low base rates of the target response included high D cell values and low A cell values, which increases Yule's Q or the exhaustive CSA index compared to the nonexhaustive CSA index.

#### *Changes in Relations Depending on Base Rates of the Stimulus (Second Event)*

A similar approach was taken to investigate the effects of high and low base rates of the stimulus (second event). To approximate conditions in which base rates of the stimulus are high, Cells A and C were generated as random numbers between 75 and 100, and Cells B and D were generated as random numbers between 0 and 25. To approximate conditions in which base rates of

the stimulus are low, the same set of contingency table values were rearranged such that Cells A and C were those generated as random numbers between 0 and 25 and Cells B and D were those generated as random numbers between 75 and 100.

Similar patterns were identified when stimulus base rates were manipulated as were identified when target-response base rates were manipulated (see Figure 5). That is, when base rates of the stimulus were high, the nonexhaustive CSA index showed a positive bias. When base rates of the stimulus were low, the nonexhaustive CSA index showed a negative bias. Again, these biases are likely due to patterns of imbalance between the A and D cell values.

#### *Controlling for Chance Occurrence of the Response–Stimulus Sequence*

Using the first set of simulation data, we computed the Pearson product-moment correlation coefficient between each contingency index and the probability of the chance occurrence of the response–stimulus sequence. This probability is calculated as the product of the response and stimulus base rates. Both Yule's Q and the exhaustive CSA index revealed very low correlations with the chance occurrence of the sequence ( $r < |.09|$ ). These near-0 correlations suggest Yule's Q and the exhaustive CSA index each control for the chance occurrence of the response–stimulus sequence. In contrast, the nonexhaustive CSA index was positively correlated with the probability of the chance occurrence of the target response followed by the stimulus ( $r = .47$ ). This positive correlation suggests that the nonexhaustive CSA index reflects chance sequencing of the target response and stimulus to a nontrivial extent. That is, the nonexhaustive CSA index is influenced by the likelihood of the target response and stimulus occurring in sequence by chance alone and thus does not adequately control for the chance occurrence of the sequence. In contrast, Yule's Q and the exhaustive CSA index do control for the chance

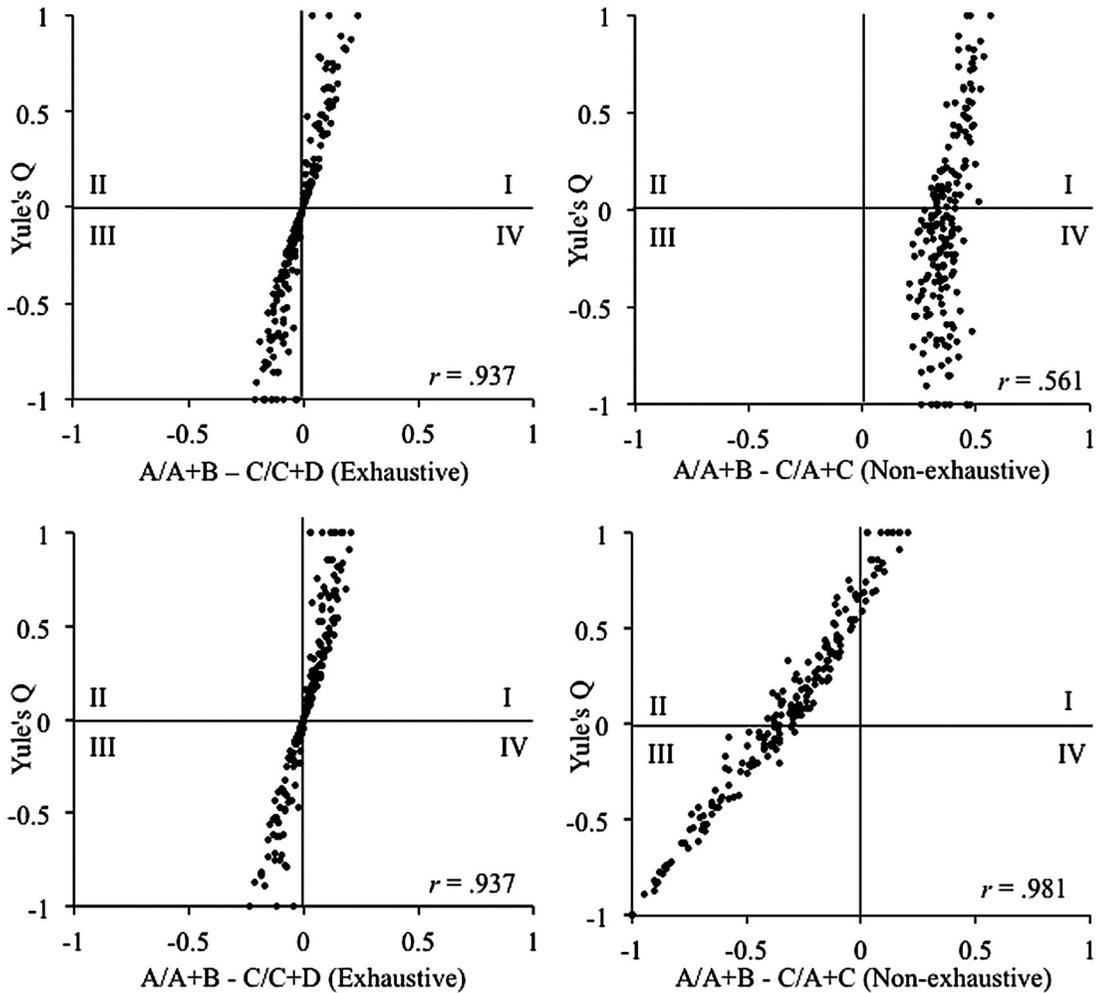


Figure 5. Relations between Yule's Q and exhaustive CSA index (left panels) and between Yule's Q and nonexhaustive CSA index (right) for high base rates of stimulus (top) and low base rates of stimulus (bottom).

occurrence of the response–stimulus sequence and thus reflect an association beyond what would be expected by chance.

### DISCUSSION

In the current paper, we identified and described two types of transitional probability comparisons used to measure response–stimulus contingencies and compared each to a well-established statistic of sequential association. For the purpose of describing response–stimulus contingencies, transitional probability compar-

isons via CSA may yield more descriptive information than single Yule's Q indices. In CSA, transitional probability differences including the D cell preserve an exhaustive coding space and appear to control for base rates of both events as well as the chance occurrence of the response–stimulus sequence. Unlike Yule's Q, the exhaustive CSA method also has the advantage of remaining interpretable even with a value of 0 in one of the contingency table cells. The non-exhaustive CSA index excludes the D cell and thus potentially avoids an unnecessary inflation of the contingency as a result of time passing

without the occurrence of either event. However, the nonexhaustive CSA index appears to be influenced by base rates of the target response and stimulus and appears to reflect chance sequencing of the two events.

### *Recommendations*

For the purpose of quantifying response–stimulus contingencies, the exhaustive CSA approach appears to provide more relevant information than a single Yule's Q statistic. The visual analysis of transitional probabilities not only facilitates the estimation of contingent, dependent, and independent relations between responses and stimuli but may also provide information on schedules of stimulus delivery in natural settings. The extent to which the exhaustive CSA index corresponds with Yule's Q suggests that it may be more appropriate than the nonexhaustive CSA approach. That is, the exhaustive CSA method produces an index of contingency that controls for the base rate of the target response, the base rate of the stimulus, and the chance occurrence of the response–stimulus sequence. Therefore, we recommend the use of the exhaustive CSA approach ( $A/A + B - C/C + D$ ) because it reflects a behavior-analytic definition of contingency and demonstrates control for base rates and chance associations.

Although it is not the focus of the current paper, an additional type of transitional probability comparisons has precedent in the literature and addresses the matter of arbitrarily defining the nonoccurrence of behavior. *Response-independent* or *background* probabilities have been calculated as an alternative contrast probability to which transitional probabilities of a stimulus given a target response are compared (McKerchar & Thompson, 2004; Vollmer *et al.*, 2001; Watson, 1997). That is, the transitional probability of a stimulus given a response may be compared to the probability of a stimulus given any randomly selected interval of the observation (*i.e.*, a simple probability of the stimulus).

Our recommendation as to whether this approach is appropriate depends on whether an exhaustive coding space is used to calculate the background probability. If the background probability includes the D cell, which appears to be consistent with previous research that has employed this method, the transitional probability difference will be closely aligned with the exhaustive CSA index (as well as Yule's Q). If the background probability does not include the D cell, the transitional probability difference will show decreased correspondence and a negative bias compared to the exhaustive CSA index (as well as Yule's Q). This negative bias would result in the identification of negative response–stimulus contingencies regardless of how the data are arranged among contingency table cells. Figure 6 depicts graphic comparisons of the exhaustive CSA index and each background probability method (exhaustive and nonexhaustive). As shown in Figure 6, even exhaustive CSA indices that approach the maximum difference score (1.0) correspond to negative nonexhaustive background probability differences. When compared to the exhaustive CSA index, the negative bias occurs because the lack of the D cell in the denominator of the contrast probability of the nonexhaustive background approach will always produce a larger probability than the contrast probability of the exhaustive background approach, and will thus produce a smaller difference score. Similar patterns were shown under high and low base-rate conditions, such that the nonexhaustive background probability method reflected negative biases and the exhaustive background probability method did not. Additional graphic comparisons that involve the background probability approach given high and low base rates of each event are available from the first author.

### *Future Research*

Additional discussion and research on contingency measurement are needed before the field reaches consensus on the most appropriate method of quantifying response–stimulus

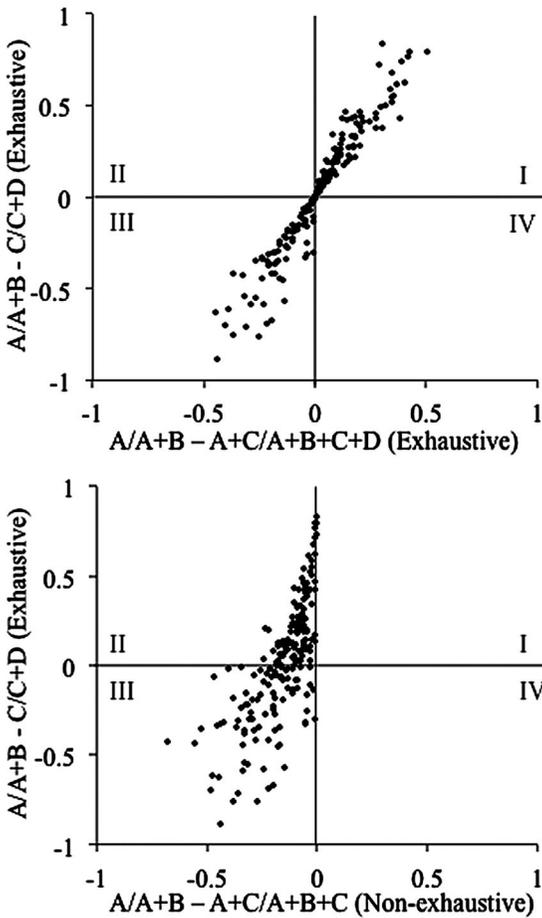


Figure 6. Relation between exhaustive CSA index and exhaustive background probability comparison (top) and between exhaustive CSA index and nonexhaustive background probability comparison (bottom).

relations in the context of natural settings. Challenges related to the definition of the nonoccurrence of behavior and the criterion for one event to be coded as following another event in sequence should be further explored. These definitions should depend on the particular response–stimulus, stimulus–stimulus, or response–response relations under investigation, and may be informed by what Skinner (1935) referred to as “natural lines of fracture” (p. 40). Measures of contingency will vary depending on the coded unit used to construct contingency tables (e.g., intervals, time windows, event pairs).

In fact, evidence suggests that the strength of association, and in some cases the direction of association, may differ depending on the precise time window selected to define response–stimulus sequences (Marion et al., 2003; Vollmer et al., 2001).

Further discussion and evaluation of what the D cell represents and contributes to behavior-analytic models of contingency also are warranted. There appears to be a lack of consensus in the field as to whether changes in D cell values (representing the absence of either event) should affect estimates of contingency strength. What the D cell represents also varies depending on whether the coded unit is time based or event based. Because time-based sequential analysis methods are more common in descriptive analyses of response–stimulus contingencies, we discussed the D cell as representing the passage of time with no occurrence of the response or stimulus. For event-based sequential analyses, however, the D cell represents pairs of events that did not include either target response or stimulus, and may be interpreted differently.

Another avenue for future research includes the establishment of guidelines for the sufficiency of descriptive data analyzed via transitional probability comparisons. In other words, how much data are needed to calculate stable estimates of response–stimulus contingencies as measured in natural settings? In the context of group design, generalizability and decision studies have been used to address this question for a variety of direct observation measures, including various types of engagement and language measures (e.g., Bruckner, Yoder, & McWilliam, 2006; McWilliam & Ware, 1994). The theory on which these analyses are based (generalizability theory; Cronbach, 1972), however, may be logically inconsistent with single-subject design (see Yoder & Symons, 2010, for discussion). The identification of strategies to evaluate the sufficiency of descriptive data used to calculate contingencies is needed to provide guidelines for data collection, analysis, and interpretation.

Finally, a caution for future research on contingency analyses in the context of the functional assessment of problem behavior is warranted. Many of the descriptive analysis studies that used transitional probability comparisons to estimate response–stimulus contingencies have focused on relations between problem behavior and subsequent stimuli. Measures of contingency strength may not be used as an indication of reinforcement effects. Contingency analyses provide only descriptions of observed response–stimulus patterns and do not permit determinations of whether one event is sensitive to changes in the other event. Strong temporal associations between problem behavior and attention, in fact, have been identified in the absence of attention-related reinforcer effects (St. Peter *et al.*, 2005). We make this point not to argue against the utility of measuring contingencies between problem behaviors and environmental stimuli but to emphasize applications aside from the identification of potential response–reinforcer relations (e.g., programming naturally occurring contingencies into subsequent experimental analyses, approximating naturally occurring schedules of reinforcement following the verification of response–reinforcer relations).

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